Exposures through common portfolio and contagion via bilateral cross-holdings among funds, banks and insurance companies


1 The views expressed in the paper are solely those of the authors and do not necessarily represent the views of the Eurosystem, the Banque de France, the Direction générale du Trésor, the Autorité des Marchés Financiers, or the Autorité de Contrôle Prudentiel et de Résolution.
Résumé

Cet article étudie la transmission d'un choc exogène sur la valorisation des actifs via un réseau financier. Nous utilisons des données individuelles de 8 308 institutions financières françaises pour construire un réseau d'entités constituées de groupes bancaires, de compagnies d'assurance et de fonds d'investissement, où chaque institution détient des actifs externes au réseau ainsi que des obligations, des actions et des parts de fonds d'investissement émises par d'autres institutions du réseau. Le réseau détient des actifs d'une valeur d'environ 11 400 milliards d'euros à fin 2016.

Notre modèle considère deux principaux canaux de contagion : les prix des titres sur le marché, et les cascades de défauts potentiels. Le canal du marché tient compte des expositions communes à des mêmes ensembles de titres, ainsi que de l'impact des pertes d'un établissement sur le prix de ses propres titres, à savoir les actions, obligations et parts de fonds émises par lui et détenues par d'autres institutions financières du réseau. Le deuxième canal reflète les pertes des institutions dues aux défauts de leurs contreparties financières. Il est augmenté par la prise en compte des prêts bilatéraux catégorisés comme « grands risques ».

Dans le cadre propre au modèle, les résultats montrent l'importance du canal du marché par rapport aux traditionnelles cascades de défaut pour la propagation de petits chocs exogènes à travers le réseau financier. Par exemple, un choc de 0,1 % sur les actifs externes (c'est-à-dire ne représentant pas des expositions bilatérales au sein du réseau), qui se traduit par une perte instantanée de 10 milliards d'euros, ne provoque pas de défaut dans nos simulations mais déclenche une contagion par le canal du marché. La robustesse générale du réseau est estimée par une simulation de stress test inversé.

Mots-clés : réseau, interconnexion financière, test de résistance macro-financier.

Classification JEL : G01, G21, G22, G23, G28.
Abstract

This paper studies the transmission of an exogenous shock on assets valuation through a financial network. We use entity-specific data on 8,308 French financial institutions to build a network of banking groups, insurance companies and individual investment funds where each institution hold external assets as well as bonds, equities and investment fund shares issued by other institutions in the network. The network holds assets worth €11.4 trillion at year-end 2016.

Our model considers two main contagion channels: securities’ market prices and potential default cascades. The securities market channel accounts for common exposures to the same set of securities and for the impact of an institution’s losses on the price of its own securities, i.e. the equities, bonds and fund shares issued by the entity and held by other financial institutions in the network. The second channel reflects institutions’ losses due to defaults by their financial counterparties, and is augmented with bilateral large exposure loans.

In the specific setting of our model, the results show the importance of the market channel – versus the traditional cascades of default – for the propagation of small exogenous shocks across the financial network. For instance, a shock of a magnitude of 0.1% to external assets (i.e. the assets that do not represent bilateral exposures within the network), which represents an instantaneous €10 billion loss, does not trigger any defaults in our simulations but trigger contagion via the market channel. A reverse-stress test simulation is run to assess the general robustness of the network.

Keywords: network, financial interconnection, system-wide stress test.

JEL classification: G01, G21, G22, G23, G28.
1. Introduction

In the years preceding the Covid-19 crisis, European financial intermediation had experienced significant changes.

Among them, while the financial system still largely relies on traditional bank credit, there has been a marked rise in non-bank financial intermediation (NBFI), defined as credit intermediation through entities and activities that are fully or partially outside the banking system (insurance corporations, pension funds, investment funds, and other financial institutions – OFIs). The collective management sector has seen significant growth over the last decade, reaching an amount of assets under management (AuM) of €13.4 trillion for investment funds and €1.2 trillion for money market funds in the euro area as at June 30, 2019. This amount has to be compared to an aggregated balance sheet size of €8.5 trillion for the insurance sector, €2.9 trillion for pension funds and €31.5 trillion for the banking sector. Both investment funds and insurance companies are playing a rising role in financing the real economy, a trend observed on a worldwide scale (see e.g. IMF 2015).

Such a structural change in the financial markets creates both new dynamics and new challenges for financial system regulators, since it is difficult to anticipate the behavior of individual institutions as well as the system as a whole in the event of financial stress. Therefore, financial authorities have engaged in efforts to review or develop their monitoring toolkits, in order to address potential risks to financial stability. Last year’s IMF Financial Sector Assessment Programme for France highlighted a need to assess and address contagion risk through financial conglomerates’ direct, indirect, and common exposures (IMF, 2019). Following the analysis initiated in Benhami et al. (2018), our article contributes to this goal.

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3 OFIs are defined e.g. in the EU Non-bank Financial Intermediation Risk Monitor, ESRB, July 2019 (see p.50). This residual category is composed of “Financial vehicle corporations engaged in securitization” (FVCs), “Financial corporations engaged in lending” (FCLs), “Security and derivative dealers” (SDDs), “Specialized financial corporations”, “Financial auxiliaries”, and “Captive financial institutions and money lenders”.
4 ECB Statistical Data Warehouse, data for Q2 2019: Total assets held by investment funds in the euro area (stock) – Series Key: IVF.M.U2.N.T0.T00.A.1.Z5.0000.Z01.E
6 ECB Statistical Data Warehouse, data for Q2 2019: Total assets/liabilities of insurance corporations in the euro area (stock) – Series Key: ICB.Q.U2.X.S128.T00.T.1.W0.S1. T.EUR
7 ECB Statistical Data Warehouse, data for Q2 2019: Total assets of pension funds in the euro area (stock) – Series Key: PFB.Q.U2.S.S129.A00.T.1.W0.S1. T.EUR
8 ECB Statistical Data Warehouse, data for Q2 2019: Total assets/liabilities reported by credit institutions in the euro area (stock) – Series Key: BSI.Q.U2.N.R.T00.A.1.Z5.0000.Z01.E
In this paper, we focus on shock transmission through a financial network of securities cross-holdings. More specifically, we construct a network of banking groups, insurance companies and individual investment funds domiciled in France where each institution hold bonds, equities and investment fund shares issued by other institutions in the network. The network also includes large bank loans to banks and insurance companies (from the “Large Exposure” reporting, see section 2.1).

The model unfolds as follows. First, an exogenous shock to the asset side of agents’ balance sheets propagates through the network via the “market channel”, through two main mechanisms:

i) marked-to-market accounting, reflecting changes in the market prices of marketable securities (bonds, equities and fund shares) on the asset side of agents’ balance sheets;

ii) the impact of the change in the total value of an agent’s assets (balance sheet) on the market value of the securities issued by said agent and held by its network counterparties. In other words, a change in an entity’s asset value should affect the market value of the equities or market debt that it issues, and holders of such securities face a corresponding marked-to-market loss in the value of their assets.

We complement the main ”market channel” with a traditional channel of default contagion. We do not calibrate recovery rates and assume, for the sake of conservatism, that in extreme cases of counterparty default, an entity may instantly lose the full value of its exposures to this counterparty. Thus, in the case of a default, the value of the shares issued by the defaulting entity is set to zero; similarly we assume a loss-given-default (LGD) of 100% on loans.

The model is close to the Debtrank methodology proposed by Battiston et al. (2012) and Bardoscia et al. (2015). The model from Battiston et al. (2012) allows asset valuation effects for banks to be taken into account, thus making an initial default unnecessary, while Bardoscia et al. (2015) extends on this work by including the possibility of accounting for further rounds of contagion. Both papers build on the seminal work of Merton (1974) by employing the so called ”liquidation equilibrium”, where firms’ equity valuation is continuously affected by the value of their assets. The disadvantage of this approach is that it mixes notions of balance sheet or prudential capital and the market value of equity whereas technically there is no direct link between the two. In the case of asset losses, banks’ balance sheet equity fully absorbs the shock in order to keep assets and liabilities balanced, while the market price of the equity does not have to reflect asset losses one-to-one. In fact, market price effects can be bigger or smaller than the balance sheet effects depending on the investors’ beliefs. Chodorow-Reich et al. (2018) support this view by analyzing the role of some financial institutions (essentially insurers and to some extent banks) as ”asset insulators”. Longer-term investors can weather unrealized losses and hold on to the temporarily depreciated assets until the recovery, while institutions relying on short-term funding are vulnerable to runs, they might have to engage in fire-sales, thus realizing losses and contributing to the downward price spiral. In other word, a variation in the value of the assets insurance companies hold does not significantly affect the market value of the securities they

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9 Deposits are not taken into account due to the paucity of data.
10 The entirety of the exposure might be lost or there might be a positive recovery rate.
have issued. The stability of insurance companies’ liabilities “protects” the value of their investors’ holdings. The authors estimate a relationship between the portfolio value of assets and insurers’ share prices. They find that during normal times, insurers’ issued equity loses only 15 cents in response to a one-dollar drop in the value of the same insurers’ assets, while during the crisis this sensitivity rose to roughly one.

Against this backdrop, our model assumes that falls in the value of banks and insurers’ assets are passed through to the value of their bonds and equity with a certain coefficient. Moreover, we assume that the total pass-through depends on the entities’ riskiness as perceived by market participants, and use extreme assumptions for this. The total pass-through is defined here as the ratio of the entities’ Critical Liability Elements (CLE, as defined as the liability quantity which, when reaching the zero lower bound, triggers an entity’s default – this concept is explained in more detail in section 2.4) to total assets. In our model, investment fund shares instantly reflect the value of their assets. Meanwhile, an entity’s default remains defined in accounting terms, according to which shocks to its assets are transferred to, and balanced by, its liabilities for which the CLE absorbs 50% in the case of insurers and 100% for other institutions.

The system is hit by an exogenous shock. As regards its calibration, we consider an ad-hoc shock of 0.1% applied to the external assets of a set of institutions. This represents an instantaneous loss of €10 million. The ad-hoc shock is useful to describe the consequences of an event that uniformly affects all institutions and thus characterizes the contribution of the network structure to shock propagation. In this paper, we performed simulations based on small shocks (0.1%) to external assets and our results tend to demonstrate that, in this context, interconnections in the French financial market did not generate default cascades. The highest aggregated losses in CLE in a first scenario (in which bonds are considered as a shock transmission channel) are less than 13% for banks, 8% for insurers and 19% for funds. These falls include both the initial shock and the contagion effects. Losses due to contagion are paramount, as they represent up to 79% of the total loss of banks, 100% of the total loss of funds and 95% of the total loss of insurance companies in this scenario. The results emphasize the distinct features of the three different categories of agents (banks, insurers and investment funds), and the role each plays in contagion.

This paper describes and stresses, in a context of rising reliance on fixed-income market funding, the importance of bonds as a market channel of contagion across financial sectors.

Moreover, this study sheds light on an important unresolved question. Derivatives exposures (on and off-balance sheet) as well as other types of liabilities and commitments are likely to amplify and/or dampen, shocks propagating through the network, with a magnitude that remains to be assessed.

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11 As in Bardoscia et al. (2015), the shock level was chosen to avoid any initial default of a node in the network, in Bardoscia et al. (2015) the shock level was 0.5%. As in Elsinger et al. (2004) and Fourel et al. (2013) with granular holding data we suggest for future studies to take a tail VaR (99.6%) of the historical distribution of asset prices to take the risk profile of agents into account in the shock calibration.

12 For a network with total assets of €11.4 trillion and €1.8 trillion of CLE.
Our paper fits into the literature on financial sector stress testing. Currently, only a few papers model system-wide stress tests with different types of agents, but the field is growing rapidly. Existing papers on stress tests mostly use simulated data (with one or several representative agents in each sector) or use real data and focus on one financial sector. The paper that models a financial system in the most comprehensive way is by Aikman et al. (2019). The authors propose a model with a set of representative agents (commercial banks, broker-dealers, insurance companies and pension funds, hedge funds, money-market funds and other investment funds) that interact in several asset, funding and derivatives markets. The authors study two main contagion channels: asset fire sales triggered by solvency and liquidity constraints or common asset holdings; and the liquidity channel driven by liquidity providers withdrawing short-term funding or by margin calls in derivatives markets. The main advantage of the model is the endogenous determination of asset prices in a general equilibrium, meaning that endogenous demand and supply determine the prices of securities. Calimani et al. (2017) also models equilibrium price determination but for a single asset traded by banks and asset managers. All institutions are connected through holdings of similar securities, and banks are further connected through bilateral lending. Halaj (2018) proposes a model with a number of simulated banks and asset managers where the main channels include funding issues, asset fire sales and losses due to direct lending between banks. None of the three papers models securities cross-holdings and risks associated with the propagation of a shock through these exposures.

Other papers analyze financial sectors individually. To cite a few, Baranova et al. (2017) simulate the behavior of investment funds and study a vicious loop between funds’ performance and their portfolio reallocation. Cont and Schaanning (2017) propose a framework to analyze the transmission of a shock through a banking system due to common portfolio holdings. Douglas et al. (2017) model the behavior of UK life insurance companies and pension funds as a response to changes in asset prices and highlight potential pro-cyclical behavior.

Overall, this paper contributes to the literature in three main respects. First, it is original in merging granular datasets to create a full network of interconnections across financial sub-sectors, whereby the network is composed of banks, investment funds and insurance companies at the national level. Funds connect to the network only through bilateral security-by-security exposures, whereas banks and insurers connect to the other French financial entities through both security-by-security and loan-by-loan exposures of banks to insurance companies, provided the loan amount exceeds a given threshold. Second, it models common exposures and shock propagation through bilateral securities holdings via a market channel, as a basis for computing a range of network contagion metrics, building on the existing literature. Third, it reviews and investigates in detail the accounting rules and business models of the types of agents under consideration.

This work could be extended. The current results point to three possible improvements in the model. First, the introduction of granular shocks may give a more realistic idea of the size of the effect. Second, in line with Chodorow-Reich et al. (2018), the sensitivity of an institution’s securities prices (bonds and equities) to shocks to its asset side could be precisely measured and the estimates could be integrated into our model (in the current paper, we use ad-hoc assumptions). A sensitivity study could measure the empirical insulation power of insurers and banks for both bonds and shares. Furthermore, our intuition is that the asset insulation power should be linked to the level of CLE, and
this relation could take the form of either a function or a threshold. Finally, derivatives (on and off-balance sheet), as well as other types of liabilities and commitment, may play a role in the size of the contagion effect and a follow-up study on the French market could be interesting. An extension to include cross-border linkages, although desirable, does not seem easily achievable in the near future due to legal data access issues.

The paper is broken down as follows: section 2 describes the data sources and the network construction process; section 3 describes and motivates the modelling assumptions; section 4 describes the network contagion measures that are relied upon; section 5 gives an overview of the characteristics of the two scenarios tested; section 6 discusses the reach of the empirical results; section 7 is dedicated to a reverse stress test; section 8 concludes.

2. Data

2.1 Construction of the database

To conduct this study, we merge several databases on investment funds (collective investment undertakings, CIUs) domiciled in France, as well as on French banks and insurance companies.

The first set of data provides the composition of the market securities portfolios (ISIN-by-ISIN) of these three types of institutions: the Banque de France Funds Collection identifies the (line-by-line) composition of investment funds’ portfolios; the PROTIDE report provides detailed information on banks' holdings of market securities; finally, for insurance companies, the template S.06.02 introduced by the Solvency II Directive gives the detailed list of assets included in the balance-sheet. We distinguish between securities held by insurers for euro-denominated contracts and those held through unit-linked accounts. The scope of securities held includes equities, bonds and fund shares issued by entities of all (domestic and foreign) nationalities belonging to the financial sector but also to the non-financial sector, excluding sovereign securities for which we do not model the sovereign counterparty risk. We add granular loan information from the banks’ Large Exposure reports at a consolidated level, on loans larger than €300 million (or 10% of the reporting bank’s capital).

See the Commission Implementing Regulation (EU) 2015/2450.

NB: Unit-linked (UC) insurance contracts are not included in our study to avoid double counting with funds (unit-linked contracts are invested in funds – note, however, that French unit-linked contracts can be invested in non-French funds, which are outside our network).

Article 392 of the Capital Requirements Regulation (CRR – Regulation (EU) 575/2013) defines a “large exposure” as an exposure, before the application of credit risk mitigation (CRM) measures and exemptions, equal to or higher than 10% of a bank’s eligible capital vis-à-vis an individual client or group of connected clients.
The second major set of data corresponds to the balance sheet items of the financial institutions. For the funds, we retain the variables of total assets (total balance sheet) and total net assets (net asset value per share multiplied by the number of shares outstanding) from investment funds’ data collection on assets and liabilities statistics\textsuperscript{16}. The banks’ balance sheet items include total assets and Common Equity Tier 1 (CET1)\textsuperscript{17} regulatory capital from COREP\textsuperscript{18}. The balance sheet variables for insurance companies cover total assets, own funds, prudential capital requirements such as the minimum capital requirement (MCR)\textsuperscript{19} and risk margins from Solvency II reporting.

The final database obtained by aggregating all the above-mentioned data thus provides a detailed view of the different types of interconnection within the French financial system.

First, by matching the securities identified on the asset side of a French financial entity’s balance sheet (its asset portfolio) with the securities issued by the other French financial entities (and therefore present in the liabilities of these entities), we obtain the French financial sector’s exposures through bilateral cross-holdings of market securities. It is thus possible to describe a network where a Bank $B$ holds units of Fund $F$ that in turn holds securities (shares and/or bonds) issued by Insurer $I$, which itself may have invested in Bank $B$’s securities and Fund $F$’s units. The blue arrows in Figure 1 represent these cross-exposures.

\textit{Figure 1: Direct and indirect interconnections}

\begin{center}
\begin{tikzpicture}
% TikZ code for the diagram
\end{tikzpicture}
\end{center}

\textit{Note: Cross-exposures are represented by the holdings of the various securities: equities, bonds and fund shares. $B \rightarrow I$ means that Bank $B$ holds a security issued by Insurance company $I$ (which is therefore $I$’s liability). NFC stands for non-financial corporations and “Others” includes public administrations, households and other financial intermediaries.}

\textsuperscript{16} Regulation (EU) No 1073/2013 of the European Central Bank of 18 October 2013 concerning statistics on the assets and liabilities of investment funds (ECB/2013/38). The description of the data collection framework can be found, in French, on the Banque de France website.

\textsuperscript{17} Banks’ CET1 is further defined in section 2.4.

\textsuperscript{18} The COmmon solvency ratio REPorting framework (COREP) was introduced by the Committee of European Banking Supervisors in 2006 to promote convergence in bank solvency ratios reporting within the EU and is now updated by the European Banking Authority (EBA).

\textsuperscript{19} The insurance companies’ MCR is further defined section 2.4.
In addition, the detailed knowledge of all securities held by French financial institutions makes it possible to understand indirect exposures between institutions, through the presence of similar securities in the portfolios of different financial entities. These joint holdings cover all the securities issued by companies, be they French or foreign, financial or non-financial; they are represented by the black arrows in Figure 1.

The data work needed to build a network as well as the computing power required to analyze common portfolios and run simulations are burdensome. Therefore, the proposed analysis is limited to a snapshot of exposures as of December 31, 2016.

2.2 Separation of the banking and insurance activities of a group holding

In our dataset, we have entities (insurers and banks) which are part of a same financial group.

On the liability side, for the banks’ CET1, we verified that all French banking groups use the same approach for their stakeholdings in insurance companies. They all use the so-called “Danish compromise”: they do not deduct their stakes in insurance companies but apply a 250% weighting to those exposures. Furthermore, in line with our modelling approach, those exposures are booked at fair value on the asset side.

On the asset side, our dataset on banks’ holdings lacks information concerning those stakes. We therefore need to add them manually. Thanks to their 2018 annual registration documents, we identified the seven largest insurance groups which are part of a banking group: BNP Paribas Cardif (100%-owned by BNPP), SOGECAP (100%-owned by Société Générale), Crédit Agricole Assurances (100%-owned by Crédit Agricole SA), Assurances du Crédit Mutuel (80%-owned by Crédit Mutuel), Suravenir (100%-owned by Crédit Mutuel Arkéa), Natixis Assurances (72%-owned by BPCE through Natixis) and CNP (in which BPCE and LBP each own an 18% stake).

However, from supervisory data, we cannot determine the fair value of each individual stake. To model the fair value of a bank’s stake in an insurance company, we add the eligible own funds (EOF) to cover the Solvency Capital Requirement (SCR) and the risk margin. This is a proxy of how much “cash” the investor would receive in the case of an orderly liquidation of the insurance company. The addition of these data gives a more comprehensive view of the cross-holdings within our network.

2.3 Bank loans as exposures in the network

While our study focuses on contagion via the valuation of assets on the financial market, we also have information on bank loan exposures that dwarf their holdings of marketable assets. Hauton and Héam (2015) studied exposures for French conglomerates, pure banks and pure insurers and sorted the exposures into equity instruments (shares, capital investments, etc.) and debt instruments (bonds, loans, etc.), and found that the debt instruments account for 91.8% of exposures (that is, 8.2% of exposures are composed of equity instruments). We used the same data to compare the marketable
exposure that is the focus of our study and total exposure taking into account bank loans. In line with Hauton and Héam (2015), we find that a median of 90.1% of bank exposures takes the form of loans (a major component of the debt category in their paper).

2.4 Balance sheet of a financial institution

In our study, the financial system is composed of three types of agents: funds, banks and insurance companies. The balance sheet of each actor is modeled as follows:

On the asset side, agents hold:

1) Marketable securities (including listed equities, bonds and fund shares) issued by other French financial institutions (i.e. recorded as liabilities by these institutions). These securities are known ISIN-by-ISIN and thus define the internal exposure to the network. We denote by $B$ the total value of all these securities on the asset side of the agent considered.

2) Bank loans to another French financial institution in the network that exceed €300 million or 10% of the originating bank’s capital, denoted by $B_L$.

3) Other marketable securities known at the ISIN level, including securities:
   a. issued by foreign financial agents;
   b. issued by non-financial French and foreign agents.

4) Other assets (derivatives, cash, other loans, French and foreign sovereign securities, and residual assets).

The total value of the assets mentioned in 3) and 4) is referred to as "assets external to the network", $A_x$. In other words, the external assets of a French financial entity correspond to all other assets except the marketable securities and bank loans of the domestic financial network. Denoting total balance sheet assets as $TA$, $TA = B + B_L + A_x$. 

On the liability side of banks and insurers, we identify the following liabilities to evaluate the solvency of agents:

1) **Critical Liability Elements** (labeled as CLE). The introduction of this element is intended to homogenize the terms used in the accounting rules of different institutions:
   a. For banks, CLE corresponds to Common Equity Tier 1 (CET1). This accounting concept refers to the core of the banks’ own funds, and CET1 mainly comprises the banks’ equity.\(^{20}\)
   b. For insurance companies, CLE corresponds to the prudential concept of the capital\(^ {21} \) required to cover the minimum capital requirement (MCR) as defined by the Solvency II Directive.\(^ {22} \)

2) market debt, consisting of bonds issued by banks and insurance companies;

3) other liabilities (e.g. loans from other banks).

For funds, liabilities are composed of:

1) CLE, corresponding to total net assets (denoted by TNA, which is defined as the product of the net asset value (NAV) per share and the number of outstanding shares – note that despite its name, the TNA is a liability item for the fund).\(^ {23} \) Fund units are held either by other network agents or by investors outside the network.

2) If applicable, other liabilities (such as bank loans).

\(^{20}\) To which are added undistributed profits and unrealized gains and losses, minority interests, reserves and funds for general banking risks (unallocated provisions). From this are deducted goodwill, holdings in CET1 held in financial companies with a franchise, deferred tax assets dependent on future profits and shortfalls in provisions.

\(^{21}\) An insurer’s own funds (FP) are defined as excess of assets over liabilities and classified into three major categories (or Tiers). Only Tier 1, Tier 1 restricted and Tier 2 cover the MCR. These elements are defined in the Delegated Regulation (EU) 2015/35 (Articles 69 to 77). Tier 1 consists in particular of the capital in released ordinary shares, the surplus funds of the reconciliation reserve (part of the excess of assets over liabilities unclassifiable in a Tier). Tier 1 restricted consists mainly of paid-up preferred shares and subordinated debt. Tier 2 consists mainly of unvested ordinary share capital, unvested preferred shares and non-paid subordinated debt.

\(^{22}\) This is defined in Article 248 of the Delegated Regulation (EU) 2015/35 as

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\text{MCR} = \min ( \max [\text{MCR}_{\text{linear}}; 0,25 \times \text{SCR}] ; 0,45 \times \text{SCR} )
\]

with the constraint that it must exceed the absolute floors indicated in Article 253 of the same Regulation. The \(\text{MCR}_{\text{linear}}\) component is obtained by the linear combination of technical provisions and earned premiums. In practice, non-compliance with the MCR leads to a ban on exercising the activity following the withdrawal of accreditation. The SCR corresponds to the capital required to face a situation of losses within one year in 99.5% of cases (Value-at-Risk equivalent to 99.5% at 1 year). In our model, we consider 45% Solvency Capital Requirement (SCR) as MCR, in order to be able to model an evolution of the VaR at 99.5% at 1 year according to different stress scenarios. We do not model the temporal evolution of MCR.

Measuring funds’ leverage is actually an ongoing debate for international standard-setters and is the focus of work by market regulators within IOSCO.\textsuperscript{24} In asset management, financial leverage (through bank loans) is rather low, while synthetic leverage (through derivatives) can be significant. Yet derivatives are also largely used to hedge positions and disentangling hedges from leverage is extremely difficult (and beyond the scope of this article which excludes derivatives from the interconnectedness analysis).

The lack of information and of harmonization between different agents in our network prevents us from properly and fairly estimating fund leverage. To avoid dealing with unrealistic artificial “leverages”, we replaced the potentially inflated balance sheet size (Total Assets, TA) by the maximum value of either TNA or the sum of all known market assets in the fund’s portfolio, as robustness checks for our results.

For comparison and as a robustness test for the present model, we removed the cap on the TA/CLE ratio, taking all balance-sheet liabilities into account. Only 22\% of the observed funds have a TA/CLE ratio greater than 1.10, 99\% are below a threshold of 2.08. The aggregate results are not significantly affected.

Across all entity types, we consider that default occurs when CLE reaches zero.

The principles used for valuing securities on the asset and liability side of institutions’ balance sheet are as follows:

- On the asset side, we assume\textsuperscript{25} that financial institutions use fair value for their entire securities portfolio, in accordance with the principles laid down in IFRS 13 and IFRS 9 for funds and banks, and in Solvency II for insurers. Bank loans on the asset side are recorded at their nominal value (principle of amortized costs in IFRS 9 where payments of principal are deducted as and when they occur).

- On banks’ liability side, bonds and loans are recorded at their nominal value (principle of amortized costs in IFRS 9 where payments of principal are deducted as and when they occur). For insurers, under Solvency II, they are recognized at market value without taking into account the credit quality of the company (Article 75). The valuation of the insurer’s technical provisions, governed by Article 77, corresponds to the expected discounted future obligations.

- On funds’ liability side, the value of issued shares (net asset value) is calculated regularly (most often on a daily basis) to reflect the value of the securities portfolio.


\textsuperscript{25} We consider this assumption consistent with our model. With IFRS 9, some securities could be at amortized cost if they are hold “Solely for Payments of Principal and Interest”. However, in case of a shock, the financial institution could be forced to switch from the business model of amortized cost portfolio to a fair value portfolio.
The shares issued by listed banks and insurance companies constitute the market capitalization of the considered entity. This value is different from the regulatory capital recorded as a liability in their balance sheet. The model distinguishes the value of the accounting capital (regulatory capital) of banks and insurance companies from that of the shares they issue in the market.

2.5 Descriptive statistics: holdings data

2.5.1 Global view of the financial system

To understand how the different financial sectors are exposed to each other and to other sectors, we use aggregated data. Figure 3 gives an overview of these exposures for all entities in each sector in Q4 2016, as a percentage of total assets. Sovereign debt holdings are included for general information even though we do not use this information in our analysis.
Figure 3: Securities holdings of banks / funds / insurers in our network, as a % of their total assets

Figure 3 shows that the network has significant holdings of bank securities, mainly through insurance companies that invest 40.9% of their market assets in French bank securities. Insurance companies are not owned to a very great extent by other French financial actors. Most French insurers are not listed in the stock market and do not issue market debt since their main source of funding comes from underwriting by policyholders. French fund shares account for 15.7% of French insurers’ market assets, 18.7% of French funds’ market assets and 17.8% of French banks’ market assets. Finally, French insurers seem to have the biggest home bias in their market asset portfolio, investing only 25.8% in foreign securities.

2.5.1 Description of network members

Holdings data from banks and insurance companies are reported at the entity rather than the consolidated level. For our analysis, we aggregate holdings at the banking or insurance group level, assuming that this group level bears the losses. After the consolidation, we end up with 14 banking groups and 32 insurance groups, which represent 99% and 90% of the French banking and insurance system in terms of assets respectively. As regards funds, we take all the funds registered in France.

26 “Home bias” is the tendency of an investor to focus his portfolio on domestic securities.
(i.e. regulated under the French law) for which we have information: 8,262. In the following sections, we describe each type of population through the presentation of:

- the average balance sheet composition of the representative agent per category;
- the relative amount and dispersion of known assets, CLE\(^{27}\) and total assets;
- the exposure of each population to other members of the network.

2.5.1.1 Banks

A) Balance sheet composition

The next figure shows banks’ average balance sheet composition.

Although banks’ main business is to grant loans to customers, they also hold significant amounts of bonds and derivatives. On the liability side, large banks mainly have deposits but smaller banks with more specific business models have around 50% of their liabilities in the form of bonds. Total equity capital, which we use as a robustness proxy in our model, accounts for 6% of total assets.

\(^{27}\) Recall that the concept of Critical Liability Elements, defined in more detail in section 2.4, relates to a quantity that proxies the default of an entity: when the CLE reaches the zero lower bound, the institution is said to default.
B) Dispersion and exposures

For banks, the median ratio of total assets to CLE is 22 while the median value of the ratio of known market assets held by banks to CLE is 2.5. For the median bank, the known market assets would need to be almost halved for that bank CLE to be wiped out. This rule-of-thumb is valid for all the banks in our sample as the dispersion is limited.

Figure 5: French banks’ total assets over CLE (left) and known market assets over CLE (right)

The next figure shows that banking groups essentially hold securities issued by other banks (especially loans). Moreover, exposures vary markedly between groups, with some groups being significantly more exposed. Average exposure to the banking sector is 16.6% of CLE whereas median exposure is four times smaller. Exposures in terms of total assets are less than 1%.

Figure 6: Exposure of French banks to other members of the network through financial securities (left), stock market securities and funds shares (middle), and loans (right)

Note: CLE (Critical Liability Elements) is defined in detail in section 2.4; IC stands for Insurance Companies. Loans are limited to large exposures.
2.5.1.2 Insurance undertakings

A) Balance sheet composition

The next figure shows the average balance sheet composition of insurance companies present in the network.

Insurers’ business depends on the insurance contracts they sign with customers. We provide here a prudential balance sheet according to Solvency II. They collect premiums, which they invest mainly in bonds and fund shares (for a fee, on behalf of their clients). The insurance companies are liable for the insurance contracts and they model technical provisions to be able to cover the claims they will face in the future.

Note that contrary to the other categories of agents, insurers’ CLE is not straightforwardly visible on the graph, as the MCR surplus is a prudential concept and is only a fraction of the “other liabilities” item.

For insurance companies, the median ratio of total assets to CLE is 12.2 while the median value of the ratio of known market assets held by insurers to CLE is 6.7. For our median insurance company, the known market assets would need to lose 15% for the CLE to be wiped out.
B) Dispersion and exposures

*Figure 8: Insurers’ total assets over CLE (left) and known market assets over CLE (right)*

The next figure shows that the insurance groups are significantly exposed to banks and funds. Their exposure to banks transits mainly through bonds holdings.

*Figure 9: Exposure of French insurance undertakings to other members of the network through financial securities (left) and fund shares and stock market securities (right)*
2.5.1.3 Investment funds and money market funds

A) Collective investment schemes market distribution

The ECB’s Guidelines on monetary and financial statistics (ECB/2014/15) requires that national central banks report investment funds statistics broken down into six broad categories, based on the nature of their investment: Equity funds, Bond funds, Mixed funds, Real estate funds, Hedge funds and Other funds (see Article 19).²⁸

As there are very few French hedge funds and given their limited AuM, this subsector was merged with the “Other investment funds” category. Last, note that, although Central Banks’ statistics allocate money market funds to the “monetary financial institutions”, we analyze them as a specific category of the funds’ sector in this study.

Figure 10 presents the repartition of French funds’ AuM across the six resulting categories as of December 2016:

²⁸ See also the ECB’s Manual on investment fund statistics, dated Dec. 2017
B) Balance sheet composition

In the following graphs, we show the average composition of funds’ balance sheets by type of fund.

*Figure 11: Funds’ average balance sheet composition by fund type*
For all types of funds, the Total Net Assets (i.e. the cumulative value of the outstanding shares issued by the fund) is by far the largest liability item. On the asset side, unsurprisingly, the composition of the portfolio depends on the strategy assigned to the fund. For instance, the assets of money market funds (MMFs) is almost exclusively composed of bonds, shares of other MMFs, and bank deposits.

C) Dispersion and exposures

For funds, the median ratio of total assets to CLE is one while the median value of the ratio of known market assets held by funds to CLE is 0.99. For the vast majority of funds, the net asset value per share reflects almost exactly the variations in the value of the portfolio of assets, and this portfolio is composed almost exclusively of marketable securities (stock, bonds, fund shares) precisely identified with our databases. However, the dispersion is important, with several very high ratios (above 10). Leaving aside possible reporting errors, these outlying data points can be traced back to the valuation standards applying to (small) run-off funds or to the accounting treatment of some derivatives (in particular the total return swaps and the currency hedges).

Figure 12: Funds’ total assets over CLE (left) and known market assets over CLE (right)

Our general conclusions in the case of small (0.1%) shocks to external assets are not significantly altered when the ratio is capped to remove outliers, as shown in section 9.4.

29 For the bulk of our sample we have detailed information on the full portfolio of the fund.
The next figure shows that within the financial sector, funds are mostly exposed to other funds\textsuperscript{31}.

\textit{Figure 14: Exposure of French investment funds to other members of the network through financial securities (left) and stock market securities (right)}

\textsuperscript{30}Outliers are defined as observations that fall below $[Q_1 - 1.5 \times IQR]$ or above $[Q_3 + 1.5 IQR]$, where $Q_1$ is the first quartile, $Q_3$ the third quartile and $IQR$ the interquartile range.

\textsuperscript{31}As our focus is on cross-holdings, we do not model the risk stemming from derivatives, as doing so would require extensive work and data. Nevertheless, we have fund-by-fund information on the notional and market values of derivatives positions, as shown in Appendix 9.2.
2.5.2 Common portfolio holdings

In addition to direct bilateral holdings, entities may be indirectly linked to each other through similarities in the composition of their portfolios. In this case, a shock to a particular group of assets affects all the entities holding these assets. Knowing how an entity's assets are correlated with the assets of the rest of the system or of a group of entities allows a finer modeling of a shock's propagation through the system or the impact of asset fire sales in a stressed institution.

Following Fricke et al. (2017), we define an overlap weighted matrix for two entities $i$ and $j$ as:

$$
\frac{\sum_s M_{is}M_{js}}{\sqrt{\sum_s M_{is}^2 \sum_s M_{js}^2}},
$$

where $M$ is a matrix of weights in each security of each entity: $M_{is} = \frac{w_{is}}{\sum_s w_{is}}$ and $W_{is}$ is a holding amount in euros of node $i$ in security $s$.

3. Model

3.1 Channels of shock propagation

The model aims to describe a shock transmission through market prices, taking into account frictions as much as possible (possible shock amplification or absorption mechanisms). It also aims, like a vast body of literature on networks, to model the possibility of default of a considered entity and its subsequent impact on the rest of the network.

Shock propagation through a network is thus modeled through two distinct channels:

A) A contagion channel through asset prices (called the "financial market channel"):

The purpose of the financial market channel is to model contagion through marketable securities holdings. Figure 15 illustrates the impact of a fall in the market value of security 1 held by two entities $i$ and $j$. This fall:

- directly affects the common holding of security 1 (grey arrow): the fall in value of security 1 is reflected for institutions $i$ and $j$, all things being equal, by a proportional reduction in their balance sheets. Then, in line with the efficient market hypothesis, this decline in the value of total assets affects the market value of the entities concerned, and therefore reduces the price of the equities and bonds that they have issued.

- indirectly affects entities holding securities issued by institutions $i$ and $j$, which hold security 1 and have lost value due to the initial shock (green dashed arrow). Thus, entity $j$ is affected twice though its direct holding of asset 1 and through its holding of securities issued by institution $i$. 


B) An "accounting" channel of contagion (through defaults):

At the same time, losses resulting from shocks are also absorbed by the entity via an accounting channel: a drop in the market value of the asset though direct and indirect channels is absorbed by the Critical Liability Elements, CLE (red arrow).

Modeling the accounting channel allows us to take into account the potential default of institutions that make up the network.

Figure 15: Shock transmission channels

A default occurs when CLE is equal to zero, i.e.:

- for an investment fund: when its total net assets become nil;
- for a bank: when its CET1 regulatory capital becomes nil;\(^{32}\)
- for an insurer: when its eligible own funds no longer cover the minimum capital requirement (MCR).

---

\(^{32}\) Bank minimum regulatory capital is fixed at 4.5% of RWA plus the Pillar 2 buffer. According to the Bank Recovery and Resolution Directive (BRRD – \textbf{2014/59/UE}), a bank is under resolution as soon as this threshold is crossed.
If a default occurs, holders lose 100% of their exposure in equities, bonds, fund shares and bank loans. We are well aware that the loss can be mitigated by the recovery rate (RR). Altman et al. (2004) review the literature on default recovery rates and clearly point out the drawbacks from Merton-type models where the value of the firm is the core of the RR. Altman et al. (2004) point to models where RR is sensitive to the loan characteristics and macroeconomic conditions. Based on a review of the literature, RR ranges from 30% for unsecured loans to 73% for senior secured loans. Historical data recorded in Rajan and Ramcharan (2016) point toward a median RR for the 1920s US bank sample of around 52%, and this RR is significantly improved after 1932, when the US federal government created the Reconstruction Finance Corporation (RFC) to help stem the wave of bank suspensions by lending directly to troubled banks. In our data set, unsecured loans represent only 7% of the total loan volume. For the sake of conservatism, we model a nil RR.

3.2 Model of shock propagation in the network

3.2.1 Contagion mechanism

We define the bilateral matrix of exposure where node \( i \) is exposed to node \( j \) at time \( t \) by the amount: \( B^i_{t,j} \). The contagion mechanism is similar whether we consider equities, bonds or fund shares. Therefore, for simplicity, we use a generic matrix \( B \). In practice, one matrix per security type is used to take into account the different sensitivity of the market. \( TA^i_t \) is total assets, \( Ax^i_t \) external assets, and \( CLE^i_t \) the Critical Liability Elements of node \( i \) at time \( t \). As suggested by Chodorow-Reich et al. (2018), we can calibrate with time series or expert judgment a link between the total assets accounting value in the balance sheet and the market value of that same company’s equities and/or bonds, writing the sensitivity parameter that can depend on whether the asset is a bond or an equity. We assume that market prices react with a linear coefficient \( \alpha_{\text{stress}}^{\text{type}} \) to the ratio of total assets over CLE and suggest different sensitivity scale parameters in section 3.3 that depend on the stress conditions of the economy and the security type. We consider an initial shock of \( \delta^j \% \) to the external assets of node \( j \):

\[
Ax^j_{t=1} = Ax^j_{t=0} (1 - \delta^j)
\]

This initial shock then propagates via the financial market channel:

\[
\forall i \neq j, TA^i_{t=1} = TA^i_{t=0} - \frac{P^i_{t=0} \delta^j \alpha_{\text{stress}}^{\text{type}} Ax^i_{t=0}}{CLE^i_{t=0}}
\]

33 In our model, contagion via bank loans only occurs in the event of a default, where the value of the loan is withdrawn from the assets of the bank exposed to the defaulting entity.

34 We consider that on the asset side, equities and bonds are "marked to market", meaning that the accounting value reflects the market value quasi-instantaneously. This roughly corresponds to the "held-for-trading" asset category.
This initial shock affects the CLE via the accounting channel:

\[ CLE_t - CLE_{t-1} = TA_t - TA_{t-1} \]

In the literature, the bilateral exposures matrix \( B \) is generally of a much smaller order of magnitude than that of total assets or even CLE. However, this is not the case for funds, so it is necessary to recalculate the matrix \( B \) between each iteration. Otherwise, the value of an exposure could exceed the total net assets of an institution in some cases:

\[ \forall i, j, i \neq j, B_{t+1}^{i,j} = B_t^{i,j} (1 + \alpha^\text{type}_t \frac{TA_t^i}{CLE_t^j} \frac{TA_t^j - TA_{t-1}^j}{TA_{t-1}^j}) \]

We define the augmented leverage matrix:

\[ \forall i, j, i \neq j, \Lambda_t^{i,j} = \alpha^\text{type}_t \frac{TA_t^i}{TA_t^j} B_t^{i,j} \]

We can then write the market contagion steps as:

\[ \frac{TA_{t+1}^i - TA_t^i}{TA_t^i} = \Lambda_t \frac{TA_t^i - TA_{t-1}^i}{TA_{t-1}^i} \]

If at time \( t \) a node \( j \) defaults, as we consider a nil recovery rate for its equities, bonds, share funds or loans, the default contagion becomes:

\[ \forall i \neq j, TA_{t+1}^i = TA_t^i - B_t^{i,j} \]

An illustrative example of contagion is provided in Appendix 9.1.

### 3.2.2 Convergence of the algorithm

In Bardoscia et al. (2015), the amplification matrix is not updated at each contagion step. In this case, the capacity of the system to amplify the initial shock depends on the maximum eigenvalue of this matrix. In the case where the latter is smaller than one, the shocks decrease during successive iterations and the algorithm converges. When it is larger than one, at least one node will fail during successive iterations. The matrix is updated after each default until, for the updated matrix, the largest eigenvalue is less than one and no new default occurs, and the system eventually converges.

For the purposes of our study, the exposure matrix is updated at each iteration. It is therefore appropriate to iterate as long as the greatest eigenvalue of the amplification matrix is greater than one.
3.3 Discussion of the hypotheses

For banks and insurance companies, the two channels are distinct because the entities are financed by different agents (depositors, creditors and shareholders), with shareholders absorbing the shock first. A shock to the assets therefore reduces the Critical Liability Elements (CLE) of the agents, and in parallel decreases the price of the securities (shares and bonds) issued by these entities. If the absorption of shocks by the CLE via the accounting channel is conceived as purely mechanical and is considered to be one-to-one\textsuperscript{35}, the sensitivity of securities’ market value to asset losses is not determined a priori and probably varies over time. Moreover, the sensitivities of a share or bond may differ.

Regarding funds, the two channels are indistinguishable in a first approximation as long as the accounting value of the investment fund approximates the evolution of the market value of the fund’s assets. Accordingly, a decline in the value of their assets is reflected in investment funds’ net asset value, i.e. in the value of the shares they have issued.

One may note that the distinction between these two channels is useful for the temporal analysis of shock transmission. Reflecting certain market imperfections, asset-liability management adjustments may take longer than financial market valuation adjustments, which, in efficient markets, may be immediate.

A) Financial market channel

The base scenario assumes that investors devalue securities issued by agents in the same proportion as the assets held by these entities (see Scenario 1, Table 1). For investment funds, a one-to-one transmission of a shock from the asset portfolio to the fund’s share value seems straightforward, since the net asset value per share is re-valued, generally on a daily basis, according to the value of the assets making up the portfolio. For banks and insurance companies, this link is less direct and to the best of our knowledge, only one paper proposes an empirical estimation of the link between losses on the asset side and the market price of issued equities, namely Chodorow-Reich et al. (2018). The article estimates that a unit shock to the assets of an insurer led to a loss of 15% in the value of its shares in normal times, but that this coefficient increased to 100% during the 2008-09 financial crisis. For a conservative base scenario, since contagion matters particularly in times of crisis, we assume a 100% transmission of a shock to the issued securities of banks and insurance companies.

However, as a robustness test, we also test other sensitivity levels. In particular, in Scenario 2, we deactivate the contagion capacity of bonds, assuming, as in a Merton model, that as long as the value of total assets is higher than the value of debts, then debts will eventually be fully reimbursed.

\textsuperscript{35} Except for insurance companies where provisions also play an absorbing role on the liability side.
These assumptions are likely to be unrealistic, but they give a good idea of the underlying dynamics. It would be interesting to calibrate properly, for each undertaking, the issued securities’ price response to a shock on the asset side, thus extending the work of Chodorow-Reich et al. (2018). Another research focus would be to improve this model by studying the relationship between security price sensitivity and the level of CLE.

Table 1: Stock market channel: shock transmission

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bank</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>100 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Bonds</td>
<td>100 %</td>
<td>0 %</td>
</tr>
<tr>
<td><strong>Insurance company (excl. unit-linked contracts)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>100 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Bonds</td>
<td>100 %</td>
<td>0 %</td>
</tr>
<tr>
<td><strong>Funds</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fund shares</td>
<td>100 %</td>
<td>100 %</td>
</tr>
</tbody>
</table>

Other mechanisms are likely to affect shock propagation in a network, especially those related to agents’ behaviors. These mechanisms, which are not taken into account in this paper but which we aim to model in the next stage of our research, include in particular:

- **Asset fire sales by banks**: as the literature suggests (Brunnermeier and Pedersen (2009), Greenwood, Landier and Thesmar (2015)), banks target a regulatory ratio or a given level of leverage. When asset prices experience a significant drop, banks may have to re-evaluate their portfolios and sell some securities to return to their targeted level. Asset fire sales can also be induced by a liquidity shock, for example to settle margin calls or in the case of a suspension of short-term financing. Forced sales fuel the price depreciation of divested assets and amplify the initial shock.

- **Asset fire sales by funds**: some managers are constrained by a maximum value at risk (VaR). In the case of a surge in asset volatility, the manager may have to sell certain securities to meet its regulatory/internal requirements.

- **Incentives for step-in**: if a fund’s net asset value drops significantly or becomes very volatile, the head of a financial group (often a bank) may decide to intervene to provide the necessary liquidity. It must then mobilize liquidity, resulting in an increase in its RWA. Conversely, one can very well imagine that when banks’ securities lose value, the parent company will compel asset management subsidiaries to support the price by purchasing its shares.

- **Investors’ outflows away from life insurers (mass redemptions)**: the low level of participation rate in euro-denominated contracts, and concerns about the solvency of an insurer might incentivize households to lapse their life insurance contracts. A mass redemption event might also be triggered by concerns over sovereign debt (which constitutes the largest share of insurers’ portfolios).
- **Investors’ outflows away from some funds (mass redemptions):** a fall or excessive volatility in the net asset value of a fund may encourage investors to sell their shares (fund outflows), forcing the manager to liquidate a significant part of his portfolio.

**B) Accounting channel**

Although potentially secondary (as long as no default occurs), the assumptions made regarding the accounting channel deserve to be examined. On the one hand, it is assumed that, for banks, the CET1 regulatory capital absorbs 100% of the shock, an hypothesis that is prevalent in the literature (among many others, Furfine (2003), Upper and Worms (2004)). For insurance companies, the accounting treatment is more subtle and there are, to our knowledge, no research studies on the subject. According to our estimates (see Section 9.3), the capital losses are absorbed by the regulatory capital and the technical provisions on a 50%-50% basis. Concerning insurers, the split of the burden of losses (between own funds and policyholders) was defined according to simulations of the evolution in the prudential balance sheet. These simulations were performed for small losses only. This simplification is not fully representative of the complexity and non-linearity of insurers’ prudential results under Solvency II.

<table>
<thead>
<tr>
<th>Table 2: Accounting channel: shock absorption</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Banks</strong></td>
</tr>
<tr>
<td>CLE (CET1)</td>
</tr>
<tr>
<td>Other liabilities (bonds, debt, etc.)</td>
</tr>
<tr>
<td><strong>Insurance companies (excl. unit-linked contracts)</strong></td>
</tr>
<tr>
<td>CLE</td>
</tr>
<tr>
<td>o/w: provisions</td>
</tr>
<tr>
<td>o/w: own funds</td>
</tr>
<tr>
<td>Other liabilities</td>
</tr>
<tr>
<td><strong>Funds</strong></td>
</tr>
<tr>
<td>CLE (TNA)</td>
</tr>
</tbody>
</table>

**4. Measures of contagion**

In this section, we measure the contagion within the financial network resulting from market shocks. We distinguish between two categories of measures: global measures that give an understanding of the effect of the contagion on the whole system; and local measures that assess the contribution of an individual entity to the propagation of a shock.

**4.1 Global measures**

**4.1.1 Measurement of total loss in the network**

Following a shock to external assets (for all or some of the nodes in the network), we can calculate a loss in Critical Liability Elements (CLE) for the system which is equal to the sum of the losses of all the entities in the system compared to the sum of CLEs before the shock simulation.
**Total loss (%)** = \( \frac{\sum_j (\text{CLE}_j^{\text{initial}} - \text{CLE}_j^{\text{final}})}{\sum_j \text{CLE}_j^{\text{initial}}} = \frac{\sum_j \Delta \text{CLE}_j^t}{\sum_j \text{CLE}_j^{\text{initial}}} \)

This measure captures both the loss in CLE due to initial shocks and the loss in CLE due to contagion from the assets of one node to the assets of the other nodes:

\[ \text{Total Loss} = \text{Loss due to initial shocks} + \text{Loss due to contagion} \]

**4.1.2 Loss and contagion effect for the network**

For a given network, the contagion effect is measured as the difference between the total losses incurred by the network and the "immediate" losses inflicted on the network by the initial shock. All terms are expressed relative to the initial CLE of the network (measures expressed in %).

\[ \text{Loss due to contagion} \ (\%) = \frac{\sum_j \Delta \text{CLE}_j^t}{\sum_j \text{CLE}_j^{\text{initial}}} - \frac{\sum_j \Delta \text{CLE}_j^i}{\sum_j \text{CLE}_j^{\text{initial}}} \]

We can express the contagion effect as a percentage of the total loss:

\[ \text{Contagion effect} \ (\%) = \frac{\text{Loss due to contagion} \ (\%)}{\text{Total Loss} \ (\%)} = \frac{\sum_j \Delta \text{CLE}_j^t}{\sum_j \Delta \text{CLE}_j^t} - \frac{\sum_j \Delta \text{CLE}_j^i}{\sum_j \Delta \text{CLE}_j^t} \]

These measures can also be expressed by type of population (banks, insurers and funds).

**4.2 Local measures**

**4.2.1 Measuring the importance of a node for the network**

A node \( i \) is considered important for the network if it simultaneously fulfills two conditions:

- it is exposed to risky securities outside the network;
- it is well connected to the rest of the network and is therefore able to transmit significantly the initial shock (affecting its external assets).

The loss in CLE resulting from a shock to external assets applied to node \( i \) is compared to the total loss in CLE resulting from the same size of shock to every node in the network. Thus, the sum of the importance ratios of the nodes corresponds to the total loss for the network:

\[ \text{Loss due to node}_i \ (\%) = \frac{\sum_j \Delta \text{CLE}_j^{t, \text{shock node } i}}{\sum_j \text{CLE}_j^{\text{initial}}} \]

Hauton and Héam (2015) counted the number of institutions facing a loss of more than 10% in their CLE in the case of a default by node \( i \). In the work of Covi et al. (2019), the importance is called the
"Contagion index (CI)" and the difference compared with our measure stems from the fact that node \( i \) is excluded from the total loss.

For our measure, both the size of the external assets and the density of the connections with the rest of the network play a role. The measurement of the importance of node \( i \) answers the question of the weight of node \( i \) in the total loss of the network. In other words, the shock to the external assets of node \( i \) represents \( x\% \) of the total losses:

\[
\text{Importance of node}_i(\%) = \frac{\text{Loss due to a unique shock on node}_i}{\text{Total loss for a shock on all nodes}}
\]

The measure of importance could be broken down into two measures that distinguish between the risk posed by external assets and the transmission capacity of a node to the network:
1. measure of the risk of external assets exposure;
2. measure of the ability of a node to transmit a unit shock to its assets to the rest of the network.

We thus also propose a measure adjusted for the volume of external assets:

\[
\text{Adjusted importance of node}_i(\%) = \frac{\sum_j Ax^j}{nb * Ax^i} \frac{\text{Loss due to a unique shock on node}_i}{\text{Total loss for a shock on all nodes}}
\]

Where \( nb \) is the number of entities in the network.

### 4.2.2 The hub role of a node

For a given node, we can block the transmission of shocks from its assets to the assets of other nodes, which we call the hub effect of node \( i \). This means that the liabilities of node \( i \) are affected by shocks to its assets, but the securities issued by \( i \) are not affected by these shocks. This indicator measures the ability of node \( i \) to transmit shocks within the network:

\[
\text{Hub effect of node}_i(\%) = 1 - \frac{\text{CLE loss of the network without transmission by node}_i}{\text{CLE loss of the network including transmission by node}_i}
\]

For this measure, the size of the external assets of \( i \) does not play a role, only the density of the incoming and outgoing connections of this node with the other network nodes matters. However, it should be noted that in our model, the assets of an entity are composed of either network assets or external assets. As a result, the more an entity is exposed to external assets, the less it can receive incoming connections, and vice versa (endogeneity between the share of external assets and the share of incoming connections).
4.2.3 Measuring the vulnerability of a node in the network

A node is considered fragile in the network if it is tied to nodes that are exposed to external assets and that will transmit an initial shock. We thus apply shocks to the external assets of all the nodes except the node under consideration and measure the relative loss in CLE of the studied node:

\[
\text{Vulnerability of node}_i (\%) = \frac{\text{CLE loss of node}_i \text{ due to contagion following systemic shock}}{\text{Initial CLE of node}_i}
\]

For this measure, the incoming connections of the network with the considered node play a preponderant role and in particular, the size of the external assets of the nodes to which it is exposed\(^{36}\).

The measure can then be written as:

\[
\text{Vulnerability of node}_i (\%) = \frac{1}{nb - 1} \frac{\text{CLE loss of node}_i \text{ due to contagion following systemic shock}}{\text{Initial CLE of node}_i}
\]

4.2.4 The reverberation experienced by a node

Through its exposures, a node can undergo the initial shock to the external assets of node \(i\) (direct impact). This shock can then be transmitted to its liabilities and spread through the network before returning to affect its assets, and this potentially multiple times. One could also qualify this effect of reverberation as a "boomerang effect". If the node is not connected to the network or if it is connected to nodes with diversified liabilities in such a way that the network absorbs shocks to the assets of \(i\), then the reverberation effect on \(i\) will be weak. On the other hand, if the node belongs to a small community of strongly connected nodes, then the shocks will return to it regularly. We can measure this reverberation effect as follows:

\[
\text{Reverberation effect on node}_i (\%) = \frac{\Delta \text{CLE}^i_{\text{total}} - \Delta \text{CLE}^i_{\text{due to initial shock}}}{\text{CLE}^i_{\text{initial}}}
\]

\(^{36}\) Hauton and Héam (2015) counted the number of scenarios where node \(i\) faces a loss of more than 10% in its CLE. The difference between our measure and the vulnerability measure introduced by Covi et al. (2019) is that an average loss experienced by bank \(i\) is measured across all simulations.
4.2.5 Measuring the amplification of a node

Following Covi et al. (2019), we also calculate an amplification measure. When node $i$ defaults\(^{37}\), this ratio measures the loss for the network (node $i$ excluded) due to cycles $> 1$ (contagion) and the loss for the network during the first cycle (shock linked to the default) (node $i$ excluded).

$$\text{Amplification measure} = \frac{\sum_{j\neq i} \Delta \text{CLE}_1^j}{\sum_{j\neq i} \Delta \text{CLE}_1^j}$$

The next table presents a summary of local measures with their interpretations.

**Table 3: Summary of the local measures**

<table>
<thead>
<tr>
<th>Local measures</th>
<th>Calculation performed for each entity</th>
<th>Measure for an entity</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importance</td>
<td>Network loss due to the initial shock applied to a single entity.</td>
<td>Ability to transmit the initial shock. This measure depends on share of external assets, size and outgoing connections</td>
<td>In percentage of the initial CLE of the network</td>
</tr>
<tr>
<td>Adjusted importance</td>
<td>Importance measure adjusted to take into account the size of the initial external assets</td>
<td>Ability to transmit the initial shock. This measure depends on the size and outgoing connections of each node.</td>
<td>In percentage of the initial CLE of the network</td>
</tr>
<tr>
<td>Hub effect</td>
<td>Comparison of the total loss when the entity does or does not transmit the shock.</td>
<td>Ability to transmit shock. Incoming and outgoing connections.</td>
<td>In percentage of the total loss when there is shock transmission by entity $i$</td>
</tr>
<tr>
<td>Vulnerability</td>
<td>Measurement of the impact when the initial shock is applied only to other entities.</td>
<td>Incoming connections</td>
<td>In percentage of the initial CLE of the entity</td>
</tr>
<tr>
<td>Reverberation</td>
<td>Contagion effect related to the node’s own initial shock. Can be seen as a boomerang effect measure.</td>
<td>Connections to a small/very connected community</td>
<td>In percentage of the initial CLE of the entity</td>
</tr>
<tr>
<td>Amplification</td>
<td>Losses due to contagion after the default of one entity</td>
<td>Related to default size and outgoing connections</td>
<td>In percentage of the loss due to the initial default</td>
</tr>
</tbody>
</table>

\(^{37}\) In our simulations, we applied a shock to the asset side such that the CLE would drop to zero for that node $i$. 

5. Approach overview

With this study, we explore the role played by the network for shock propagation. We apply an initial shock that is small enough to guarantee that no entity from the network sees its CLE emptied after this initial shock. Therefore we apply an ad-hoc shock of 0.1% is applied to all external assets, $A_x$, of all entities.

The introduction of granular shocks that would reflect the riskiness of the known assets in a forthcoming paper would be an interesting improvement on this current work.

We study two distinct scenarios:

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock calibration</td>
<td>VaR averaged over all network nodes.</td>
<td>VaR averaged over all network nodes.</td>
</tr>
<tr>
<td></td>
<td>= 0.1%</td>
<td>= 0.1%</td>
</tr>
<tr>
<td>Shock application</td>
<td>External Assets</td>
<td>External Assets</td>
</tr>
<tr>
<td>Shock transmission</td>
<td>Market channel via marked-to-market equities, bonds and fund shares. Via loan exposures in the event of default (LGD set to 100%).</td>
<td>Market channel via marked-to-market equities and fund shares (bond prices are assumed not to react). Via loan exposures in the event of default (LGD set to 100%).</td>
</tr>
</tbody>
</table>

Scenarios 1 and 2 differ only in terms of the reaction of bond prices to the shock: in Scenario 2 the reaction of bonds is deactivated.

6. Results

6.1 Convergence

A shock of 0.1% is applied to the external assets $A_x$ of the network, using Scenario 1 and the ratios of total assets over unadjusted CLE, which is the configuration that ensures the highest level of contagion. We let the contagion algorithm run and compute for each round the maximum incremental loss in the CLE of a node (Figure 16).
6.2 Global measures

We compared the scenarios detailed in section 5:

<table>
<thead>
<tr>
<th>Table 5: Loss and contagion per scenario.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1</strong></td>
</tr>
<tr>
<td>Total loss (% CLE)</td>
</tr>
<tr>
<td>Loss due to contagion (% CLE)</td>
</tr>
<tr>
<td>Contagion effect (% total loss)</td>
</tr>
</tbody>
</table>

Therefore for Scenario 1, only 30% of the losses in CLE are due to the initial shock of 0.1% to the external assets, and almost 70% of the losses are due to the contagion effect. For Scenario 2, the contagion loss is dampened dramatically to 20%.
6.3 Results by populations and local measures

In this section, we provide results in terms of losses and local measures for both scenarios. As already mentioned, the first scenario allows a shock transmission through all securities, while the second scenario permits shock transmission only through equities. For both scenarios, an initial ad-hoc shock of 0.1% is applied to assets outside the financial network. This initial shock is equivalent to an average loss of 2% of CLE for banks, 0.1% of CLE for funds and 0.5% for insurance companies (Figure 17A). It is consistent with the size of the assets outside the financial network by category (as detailed in Figure 20).

The contagion effect is significant within the French financial network, as the average total losses amount to 5% of CLE for banks, 0.5% of CLE for funds and 2.8% for insurance companies, after the shock transmission in the first scenario. This observation shows the interconnected nature of the French financial network when we allow the shock to be transmitted through all securities. However, we logically observe a much lower loss due to contagion when we deactivate the transmission channel through bonds (Figure 17B).

When we focus on funds, money market funds face the largest losses due to contagion effects in the first scenario (Figure 18A). This can be explained by their assets, which include a large proportion of bonds issued by banks and insurance companies and investments in other money market funds. Lower average losses are also observed when we deactivate the bond transmission channel.
The next figures focus on the contagion as a share of total losses. The initial shock mostly impacts banks; thus, in relative terms, the contagion part of the total loss is lower in both scenarios (Figure 19A and Figure 19B).

In the first scenario, the relative loss due to contagion (Figure 19A) represents:
- only 28% of the total loss of banks;
- around 50% of the total loss of funds;
- 75% of the total loss of insurance companies.

As explained, these impacts are much smaller when the transmission by bonds is muted (Figure 19B).

From a supervisory perspective, it is useful to identify entities that are both vulnerable and important, or vulnerable and amplify losses. All entities in the network are depicted in a bi-dimensional chart relating importance (or amplification) and vulnerability measures. The more a node is located in the
upper-right area in the graph, the more important (shock amplifier) and vulnerable it is. Supervisory authorities could use these kinds of figures as tools to identify problematic nodes.

In the next figures, we can observe that banks tend to exhibit the highest importance measure and some of them have a high vulnerability measure. The importance measure quantifies the power of a node to transmit the initial shock to the rest of the network. As banks have a large share of external assets, the initial shock has a significant impact on their CLE. Moreover, banks’ securities are largely held by other nodes in the network, which implies, for banks, a strong ability to transmit this initial shock and consequently a higher measure of importance (Figure 20A). We also observe that the vulnerabilities of banks and insurers are significantly smaller when bonds are assumed not to transmit the shock (Figure 20B).

**Figure 20: Vulnerability vs. importance**

*Figure 20A. Scenario 1*  
*Figure 20B. Scenario 2*

However, for some types of entity, typically banks, the size of their external assets increases the magnitude of initial shocks. Thus, when measures of importance are adjusted for the relative size of external assets (adjusted importance), investment funds appear to be the most critical population in the second scenario. This observation is supported by the fact that some investment funds display a high measure of amplification (Figure 23A), reflecting the fact that some of them are strongly interconnected with other nodes in the network (they hold and are held by other nodes of the network). In Figure 21 to 23, we observe again that the bond transmission channel plays an important role.
Figure 21: Adjusted importance (in %)

Figure 21A. Scenario 1

Figure 21B. Scenario 2

Figure 22: Vulnerability vs. adjusted importance

Figure 22A. Scenario 1

Figure 22B. Scenario 2

Figure 23: Vulnerability vs. amplification

Figure 23A. Scenario 1

Figure 23B. Scenario 2
In conclusion, these results highlight the different role played by each class of institution in the network.

- The contagion effect accounts for a significant share of total losses, which underlines the interconnected nature of the French financial network.
- Due to their size and the amount of their external assets, banks play a central role in shock transmission. They display the highest measures of importance, hub effect and reverberation.
- Investment funds are very heterogeneous in terms of size and balance sheet structure (on both the asset and liability sides), which makes it harder to draw general conclusions. On average, half of their total losses are explained by the contagion effect. This shows their high level of interconnectedness and, for some of them, their high amplification capacity.
- Insurers play a lesser role in the transmission of losses. Nevertheless, they seem to be exposed on the asset side to other nodes (banks and funds) and in consequence are highly subject to contagion effects (according to their vulnerability measure).
- Finally, the results show the important role of bonds as shock transmitters.

For a deeper analysis of the explanatory power of the model, see section 9.7 in Appendix.

### 7. Reverse stress test

As defined in the BIS (2017) report on stress-testing practices, “a reverse stress test is the process of assessing a pre-defined adverse outcome for an institution, such as a breach of regulatory ratios, insolvency or illiquidity, and identifying possible scenarios that could lead to such adverse outcome”. This can lead to complex approaches when risk factors are multidimensional.

Flood and Korenko (2015) implement a grid search of scenarios assuming that risk factors observe a multivariate elliptical distribution. In a study more similar to ours, Grigat and Caccioli (2017) apply a reverse stress test to interbank networks, searching for the scenario of the smallest exogenous shock that would lead to a given final systemic loss. The authors resort to an optimization problem with some hypotheses on exogenous shocks fluctuation and time horizon. Instead of identifying risk factors – we applied exogenous shocks to balance sheet asset exposures outside our network.

A reverse stress test is implemented by gradually increasing the shock levels in order to observe defaults, or closures for a fund\(^{38}\), hence the contagion effect is no longer linear with respect to the shock magnitude.

\(^{38}\) Here, no assumptions are made regarding management. The possibility of creating side pocket funds is not taken into account. Hence, a fund is considered closed when the value of its total net assets is zero.
Figure 24: Minimum shock (in percent) required for the default of a given count of banks or insurance companies or for the closure of a given count of funds

*Note:* The shock scale in the x-axis is non-linear. As a reminder, the network is composed of 14 banking groups, 32 insurance groups and 8,262 individual funds.

When applying a uniform shock on the external assets of the network, the first default of a bank occurs at 0.65%, the first default of insurance company at 0.56% and the first closure of fund occurs with a shock of 0.38%. While shocks below 10% are sufficient to default up to 12 banks, a shock close to 50% is required to default all banks. Similarly, shocks up to 10% are sufficient to default up to 21 insurance companies, while to default up to the 32nd insurance company, jumps in shocks are necessary. As for funds, shocks below 3% are sufficient to lead to the closure of 11 funds, but shocks above 50% are necessary to close 13 funds and we are able to close less than 52 funds with shocks just below 100% of external assets.

We calibrated our ad-hoc shock at 0.1% to avoid default and measure contagion without default. Defaults start occurring with shock below 1% and with a static modelling of balance sheet no fire sales occur and we need to push the shock above 50% to default all banks and insurance companies.
Figure 25: Percentage loss of total asset of a category as a function of the shock (% of external assets)

Note: The shock scale in the x-axis is non-linear.

The aggregated total net asset of the funds that have to close corresponds only to a marginal fraction of the fund’s initial assets, even for severe losses on external assets. Yet, with shocks near 3%, banks and insurance companies that default represent more than 50% of the total asset of their respective categories.
8. Conclusion and future work

Regarding the issue of interconnectedness and its impact on financial stability, this paper adds value on three levels:

- the constitution of an extensive database of balance sheet links (market securities and some loans) between French financial institutions;
- a stylized model of contagion, on both the assets and liabilities sides; and
- the results of the simulated propagation of stylized shocks through this network of financial institutions.

A close look at the results may help to enhance our understanding of the dynamics of interconnections and consequently identify areas where the regulator should focus its attention.

The results highlight the distinct features of the three different classes of actors (banks, insurers and funds) and the role each plays in the contagion.

Insurers are the most homogeneous class and have very concentrated distributions, regardless of the indicator. This similarity also explains why, in the reverse stress test, they tend to default jointly. Moreover, they tend to be more vulnerable in the network rather than being a vector of contagion. Conversely, it is hard to analyse funds as a homogeneous actor class, as they follow very diverse strategies. This calls for complementary work, treating the funds separately by category. It is also worth noting that our imperfect approximation of funds' liabilities does not seem to have a major impact on the results, as demonstrated by a robustness test. Nevertheless, there is room for improvement in the modelling of funds' balance sheets.

Finally, banks are a particularly important class for the network in this study. This was expected, since banks are relatively large and systemic compared to the other actors in the network. This being said, their role is probably overstated by the setting of the model, since our initial shocks are applied to external assets, which make up a more significant proportion of banks' balance sheet assets than for funds and insurance companies.

The study also shows that a large portion of the interconnectedness of the financial sector is attributable to debt instruments. That does not mean that this vulnerability can be easily reduced: financial institutions must finance themselves in order to run their businesses and there is no guarantee that different liability classes would be less contagious.

Yet, our study raises unresolved questions on the interconnections in the French financial market.

- We remain agnostic on the contagion role of derivatives both on the assets and liabilities sides. This might warrant further investigation, considering their potential role in amplifying and/or dampening contagion.

39 Without taking into account large exposures, loans represent 60% of banks’ assets.
A suitable improvement to our model would be to introduce, as parameters, proper estimates of the sensitivity of a firm’s security price to shocks to its asset portfolio. The pass-through coefficients could also be used as dependent variables in cross-section regressions to assess whether they can be explained by firm characteristics. This would also be an improvement to the model created by Chodorow-Reich and used here.

Third, we applied small shocks to external assets and it would be interesting to compare those results to shocks calibrated based on the historical risk of each entity’s market portfolio.

Fourth, the question of the impact of bonds on contagion, in correlation with the development of non-bank financial intermediation (NBFI), could be explored further.
### List of abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>AuM</td>
<td>Assets under management (Funds)</td>
</tr>
<tr>
<td>CEBS</td>
<td>Committee of European Banking Supervisors (replaced by EBA)</td>
</tr>
<tr>
<td>CET1</td>
<td>Common Equity Tier 1 (Bank)</td>
</tr>
<tr>
<td>CIU</td>
<td>Collective investment undertakings (Funds)</td>
</tr>
<tr>
<td>CLE</td>
<td>Critical Liability Elements (concept introduced by the article)</td>
</tr>
<tr>
<td>COREP</td>
<td>COmmon solvency ratio REPorting (introduced by CEBS)</td>
</tr>
<tr>
<td>CRM</td>
<td>Credit Risk Mitigation</td>
</tr>
<tr>
<td>CRR</td>
<td>Capital Requirement Regulation</td>
</tr>
<tr>
<td>EBA</td>
<td>European Banking Authority (formerly CEBS)</td>
</tr>
<tr>
<td>ECB</td>
<td>European Central Bank</td>
</tr>
<tr>
<td>EOF</td>
<td>Eligible Own Funds (Insurance)</td>
</tr>
<tr>
<td>ESRB</td>
<td>European Systemic Risk Board</td>
</tr>
<tr>
<td>IC</td>
<td>Insurance Company</td>
</tr>
<tr>
<td>IFRS</td>
<td>International Financial Reporting Standards</td>
</tr>
<tr>
<td>IMF</td>
<td>International Monetary Fund</td>
</tr>
<tr>
<td>IOSCO</td>
<td>International Organisation of Securities Commissions</td>
</tr>
<tr>
<td>ISIN</td>
<td>International Securities Identification Number</td>
</tr>
<tr>
<td>LGD</td>
<td>Loss Given Default</td>
</tr>
<tr>
<td>MCR</td>
<td>Minimum Capital Requirement (Insurance)</td>
</tr>
<tr>
<td>MMF</td>
<td>Money Market Fund</td>
</tr>
<tr>
<td>NAV</td>
<td>Net Asset Value per share (Funds)</td>
</tr>
<tr>
<td>NBFi</td>
<td>Non-Bank Financial Intermediation</td>
</tr>
<tr>
<td>NFC</td>
<td>Non-Financial Corporation</td>
</tr>
<tr>
<td>OFI</td>
<td>Other Financial Institutions</td>
</tr>
<tr>
<td>O-SII</td>
<td>Other Systemically Important Institutions</td>
</tr>
<tr>
<td>RR</td>
<td>Recovery Rate</td>
</tr>
<tr>
<td>RWA</td>
<td>Risk Weighted Assets</td>
</tr>
<tr>
<td>SCR</td>
<td>Solvency Capital Requirement (Insurance)</td>
</tr>
<tr>
<td>TA</td>
<td>Total Assets (balance sheet)</td>
</tr>
<tr>
<td>TNA</td>
<td>Total Net Assets (Funds)</td>
</tr>
<tr>
<td>UC</td>
<td>Unit-linked life insurance contracts (<em>Unités de Compte</em>)</td>
</tr>
<tr>
<td>VaR</td>
<td>Value at Risk</td>
</tr>
<tr>
<td>ViF</td>
<td>Value in Force (Insurance)</td>
</tr>
</tbody>
</table>
References


Benhami Kheira, Caroline Le Moign, Dilyara Salakhova, Vinel Alexandre, Interconnections between the French asset management sector and the rest of the French financial system, HCSF publication, 2018 [link]


9. Appendix

9.1 An illustrative example of the contagion mechanisms

The mechanisms described in the previous section are illustrated below: an initial shock of magnitude $\delta$ (in percentage) to fund $i$'s external assets is transmitted through a simplified network consisting of another fund ($j$), a bank ($k$) and an insurance company ($l$).

**Figure 26: Transmission of a shock to fund $i$'s external assets**

We assume that fund $i$'s assets consist of securities issued by bank $k$ ($B^{i,k}$), shares of fund $j$ ($B^{i,j}$) and other assets outside of the network ($Ax^i$).

Other assets ($Ax^i$) are affected by a shock $\delta$ and decrease by $(\delta, Ax^i)$. Fund shares reflect the fund's asset value and adjust accordingly, thus affecting the holdings of bank $k$ ($B^{k,i}$), fund $j$ ($B^{j,i}$), and other agents outside of the network (e.g., such as French households or foreign investors).

The new value of fund $i$'s shares held by bank $k$ is based on the principles of determining the net asset value of a fund and equality of treatment of unitholders: $\left( B^{k,i}_0 - B^{k,i}_0 \cdot \frac{\Delta Ax^i}{VM^i_0} \right)$, or $B^{k,i}_0 \cdot (1 - \frac{\Delta Ax^i}{VM^i_0})$. This
means that bank $k$ is affected by the initial shock to fund $i$ in proportion to the weight of its holding of fund $i$'s shares. Similarly, the value of fund $i$'s shares held by fund $j$ is equal to $B_{0}^{i,j} \cdot \left(1 - \frac{\delta A_{x}^{i}}{VM_{0}^{i}} \right)$.

The change in value of fund $i$'s shares held by fund $j$ corresponds to the propagation of the initial shock (which concerned fund $i$'s external assets) through the network. The balance sheet of fund $j$ changes by a similar proportion and the net asset value of fund $j$ adjusts accordingly. Under our assumptions, the accounting channel and the market stock market channel are mingled for funds.

The CLE (TNA) of fund $j$ decreases by the amount $B_{0}^{0,j} \cdot \frac{\delta A_{x}^{0}}{VM_{0}^{0}}$ and the net asset value (NAV) of the fund shares adjusts to the size of the asset side. Fund $j$'s shares are held by bank $k$ but also by investors outside of the network. Bank $k$ holds $\frac{B_{0}^{k,j}}{VM_{0}^{k}}$ of fund $j$'s shares, and its assets are decreased by $B_{0}^{0,j} \cdot \frac{\delta A_{x}^{0}}{VM_{0}^{0}} \cdot \frac{B_{0}^{k,j}}{VM_{0}^{k}}$, or the new value of fund $j$'s shares on bank $k$' balance sheet is equal to $B_{0}^{k,j} \cdot \left(1 - \frac{\delta A_{x}^{0}}{VM_{0}^{0}} \cdot \frac{B_{0}^{k,j}}{VM_{0}^{k}} \right)$.

The initial shock affects bank $k$'s balance sheet twice: through holdings of shares of fund $i$ and $j$. Total losses are equal to $B_{0}^{k,i} \cdot \left(\frac{\delta A_{x}^{i}}{VM_{0}^{i}} \right) + B_{0}^{k,j} \cdot \left(\frac{\delta A_{x}^{j}}{VM_{0}^{j}} \cdot \frac{B_{0}^{k,j}}{VM_{0}^{k}} \right)$.

According to the accounting channel hypothesis, all losses on bank $k$'s assets are absorbed by its CLE (CET1) which is reduced by the amount $B_{0}^{k,i} \cdot \left(\frac{\delta A_{x}^{i}}{VM_{0}^{i}} \right) + B_{0}^{k,j} \cdot \left(\frac{\delta A_{x}^{j}}{VM_{0}^{j}} \cdot \frac{B_{0}^{k,j}}{VM_{0}^{k}} \right)$. In parallel, according to the stock market hypothesis, bank $k$’s asset losses are reflected in the market prices of the securities it issues. We assume that bank $k$’s market capitalization changes proportionally to the shock, taking into account its leverage\(^{40}\), the level of market stress: $\alpha_{k}^{T} = \alpha \cdot \frac{T A_{k}^{k}}{C L E_{k}}$. $\alpha$ reflects the market stress (different scenarios) and security type (equity share or bond). The new value of the bank's market value is equal to

$$VM_{0}^{k} - \alpha_{0}^{k} \cdot \Delta T A_{0}^{k} = VM_{0}^{k} - \alpha_{0}^{k} \cdot \left\{ B_{0}^{k,i} \cdot \left(\frac{\delta A_{x}^{i}}{VM_{0}^{i}} \right) + B_{0}^{k,j} \cdot \left(\frac{\delta A_{x}^{j} \cdot B_{0}^{k,j}}{VM_{0}^{0} \cdot VM_{0}^{j}} \right) \right\}.$$

For an insurance company, 50% of the shock to the asset is absorbed by the provisions excluding CLE (other liabilities not exposed to the network). It should be noted that provisions for CLE are booked in the reconciliation reserve.

As regards the stock market channel, the market value of the insurer's securities is reduced in proportion $\beta$ to the asset loss. The new market value of the securities issued by the insurer is:

$$VM_{0}^{l} - \beta_{0}^{l} \cdot \Delta T A_{0}^{l} = VM_{0}^{l} - \beta_{0}^{l} \cdot \left\{ B_{0}^{k,l} \cdot \frac{VM_{0}^{l} - \beta_{0}^{l}}{VM_{0}^{l}} \cdot \alpha_{0}^{l} \cdot \left\{ B_{0}^{k,i} \cdot \left(\frac{\delta A_{x}^{i}}{VM_{0}^{i}} \right) + B_{0}^{k,j} \cdot \left(\frac{\delta A_{x}^{j} \cdot B_{0}^{k,j}}{VM_{0}^{0} \cdot VM_{0}^{j}} \right) \right\} \right\}.$$

---

\(^{40}\) The leverage effect in our model plays a central role in the capacity of the network to amplify or dampen original shocks.
In the simplified network presented, the initial shock to fund $i$’s assets spreads to all entities. The contagion does not end there, since the insurance company is held (at least in part) by fund $j$, which therefore records a second round of devaluation via the discount in the price of the securities of $i$. This effect will be transmitted to bank $k$, then to insurer $l$, which will trigger successive rounds of contagion. However, because of the parameters $\alpha$ and $\beta$, each round is less severe than the previous one.

### 9.2 Funds’ notional and market derivatives positions on the assets and liabilities side

We have fund-by-fund data on the aggregated level of notional and market derivatives.

**Figure 28: Funds’ derivatives positions with (left) or without (right) outliers**
9.3 Breakdown of the loss between insurance policyholders and shareholders after a market shock, and loss absorption capacity of technical provisions

How a shock to assets affects the balance sheet of an insurance company was tackled from a prudential point of view. Solvency is considered to be a mirror of market valuation, and consequently a deterioration in solvency indirectly measures the evolution due to the stock market price. A good proxy for own funds is Value in-Force (ViF) which consists of the following components:

- the present value of future profits (where profits are post taxation shareholder cash flows from the in-force covered business and the assets backing the associated liabilities);
- the time value of financial options and guarantees;
- the frictional costs of required capital;
- the cost of residual non-hedgeable risks.

The estimation of Value-in-Force requires a cash flow projection model that takes into account liabilities stemming from life insurance contracts, taxes, regulatory reserves and an economic scenario generator (ESG). Interaction between assets and liabilities retrieves the best estimate value of the liabilities and ultimately the ViF. The breakdown of the losses between whether they are borne by customers or shareholders is determined based on the evolution of the ViF after a market shock.

In Figure 29, a -10% shock to the total value of the assets only makes the ViF decrease by 5% of the total initial value of the assets, which means that only 50% of the actual losses are borne by the shareholders.

*Figure 29: Change in ViF as a function of the shock on assets*

This can be interpreted as an absorption of the losses by policyholders. The optionality carried by insurance policies is the main driver for the non-linear loss split. However, the evolution of the ViF
after the shock is assumed to be constant with regard to the range usually considered for the applications.

If we consider a shock only to the total value of equity (Figure 30), the loss split between policyholders and shareholders still displays strong non-linearities. However, the loss share is assumed to be constant at around 36% as a reasonable approximation.

![Figure 30: Response to an equity shock](image)

Due to regulatory reserves, the impact of bond losses on own funds is less significant than the impact of equity losses. For example, the “réserve de capitalisation” acts as a buffer for policyholders against bond price variations.

### 9.4 Effect of putting a cap on the total assets over CLE ratio

Given our concerns about the inappropriateness of a simple TA/TNA ratio for capturing funds’ leverage (see section 2.4), we decided, as a robustness check, to cap the ratio of total assets over CLE to avoid distorting the results. While this concern is justified at a microprudential level, our results show that in the case of a limited shock (0.1%) to external assets, our conclusions by category of actor do not vary. To support further this conclusion, we ran the regression where the prime (’) indicates that total assets over CLE ratios have been capped for the various rounds of contagion:

\[
\frac{\Delta CLE_i}{CLE_i} = \alpha + \beta \frac{\Delta CLE'_i}{CLE'_i} + \epsilon_i
\]

We found that the intercept is not significantly different from zero and cannot reject that the coefficient $\beta$ is equal to one.
9.5 Effect of bonds on relative CLE losses

In Scenario 2, we conducted an experiment where bonds were excluded from the market channel of contagion. The impact on relative CLE losses was tested by running the regression where the prime (') indicates that bonds were excluded from the market channel for the various rounds of contagion:

\[
\frac{\Delta CLE_i}{CLE_i} = \alpha + \beta \frac{\Delta CLE'_i}{CLE'_i} + \epsilon_i
\]

We found that the effect is almost linear (with an adjusted r-square higher than 90%), significantly greater than one and tends to increase over each round of contagion before converging back to one, meaning that bonds sustain the market contagion effect in the network.

9.6 Explanatory power of network exposure regarding relative CLE losses

We tested how the level of an entity's market portfolio exposure to the network drives its relative CLE loss in each round. We ran the following regressions:

\[
\frac{\Delta CLE_i}{CLE_i} = \alpha + \beta \frac{Expo to network_i}{CLE_i} + \epsilon_i
\]

The dependent variable in this regression is the increment in CLE losses between rounds.

For each round that follows the initial shock, the coefficient \( \beta \) is positive and significant. The adjusted r-square ranges from 14% to 31%, depending on the round. There is no trend in the evolution of adjusted r-square while the coefficient \( \beta \) decreases with every round, which tends to confirm the idea that it is more the position within the network that drives the losses than the level of exposure to the network. This result remains consistent but with a lower order of magnitude when bonds are excluded (scenario 2).

9.7 Explanatory power of two-way network exposure and centrality to our local measures

For each local measure (adjusted importance, vulnerability, hub effect, reverberation and amplification), we first ran a pairwise correlation:
To complete the analysis, we tested the full regression:

\[
Local\ measure_i = \alpha + \beta \frac{Ax_i}{CLE_i} + \gamma \sum_{j \neq i} \text{exposure to } i \frac{\text{exposure to network}_i}{CLE_j} + \delta \frac{\text{Exposure to network}_i}{CLE_i} + \theta wBC_i + \rho \text{indegree}_i + \mu \text{outdegree}_i + \tau \text{closeness}_i + \varepsilon_i
\]

where \( wBC \) is the weighted betweenness centrality of node \( i \). After testing this regression, we restricted the regression and searched to select only one explanatory variable. All methods we used based on adjusted \( R^2 \), Akaike Information Criterion and residual mean square errors led to the same selection. With the first full regression, we wanted to determine whether our model can be fully explained by network data, and then for each local measure we tried to provide some intuition on the network variable that best explains the observed variance.

### 9.7.1 Explanatory power of external assets and network exposures to a node regarding its adjusted importance

We tested how the relative level of network market portfolio exposures to a node, and the relative level of the node’s external assets drive that node’s adjusted importance. The full regression has a poor explanatory power, so, for a regulator, our model can provide additional insight into the adjusted importance of an institution in a network, compared with looking only at portfolio data, balance sheet aggregates to compute network measures.
9.7.2 Explanatory power of exposure to the network regarding a node’s vulnerability

We tested how the relative exposure of a node’s market portfolio to the market drives that node’s vulnerability. We ran the full regression.

Although we found some positive and significant parameters, this approach was not conclusive, which suggests more investigation is required into the role of a node’s position in the network in determining its vulnerability. As an illustration, we scatter plotted vulnerability against the explanatory variable that we selected with our search method:

*Figure 32: Explanatory power of network asset exposure with regard to vulnerability*

9.7.3 Explanatory power of the network centrality measure regarding a node’s hub effect

We ran the full regression for the hub effect, which explains 76% of the variance. With our search algorithm, forcing a selection of only one explanatory variable, we limited our regression to:

\[
Hub_i = \alpha + \beta \frac{\sum_{j \neq i} \text{exposure to } i}{\sum_{j \neq i} CLE_j} + \varepsilon_i
\]

With this regression, we were able to explain 69% of the variance with a significant and positive sign on parameter \( \beta \). This provided us with the intuitive information that the exposure of the network to a node, in our network, has good explanatory power for the hub effect. Interestingly, this is not the case for the node \( i \)’s exposure on the asset side to the network, even one would expect that both the
assets and liabilities’ relation to the network would play a role in determining the hub effect. In our network configuration, only liabilities play a significant role.

### 9.7.4 Explanatory power of exposures regarding a node’s reverberation effect

We ran the full regression for the reverberation effect, which explains 68% of the variance. With our search algorithm, forcing a selection of only one explanatory variable, we limited our regression to:

\[
Reverb_i = \alpha + \beta \sum_{j \neq i} \text{exposure to } i \frac{\text{CLE}_j}{\sum_{j \neq i} \text{CLE}_j} + \epsilon_i
\]

With these dependent variables, we are able to explain 61% of the variance with a significant and positive sign on parameters \( \beta \). This again provided us with the same intuitions as for the hub effect.

### 9.7.5 Explanatory power of portfolio overlapping between two nodes regarding their measures

We first defined, for a given measure (vulnerability in our case), the similarity between each pair of institutions i and j:

\[
similarity_{i,j} = 1 - \frac{|x_i - x_j|}{|x_{max} - x_{min}|}
\]

We then tested our hypothesis that the overall market portfolio composition can be extrapolated from a more limited set of information, the network portfolio. We ran the regression:

\[
similarity_{i,j} = \alpha + \beta OWM_{i,j} + \epsilon_{i,j}
\]

where the overlapping metrics \( OWM_{i,j} \) are defined in section 2.5.2. We did not find any significant regression and thus would recommend separating the analysis of market exposure risk from the network market assets contagion study.

### 9.7.6 Explanatory power of our local measures regarding a bank’s OSIIs score
We tested how OSIIs\textsuperscript{41} scores for the 14 French banks can be explained either by our local measures or by banks’ balance sheet aggregates and network centrality:

\[
Score_i = \alpha + \beta \frac{Ax_i}{CLE_i} + \gamma \frac{\sum_{j \neq i} \text{exposure to } i}{\sum_{j \neq i} CLE_j} + \delta \frac{\text{Exposure to network}_i}{CLE_i} + \theta wBC_i + \rho \text{pindegree}_i \\
+ \mu \text{outdegree}_i + \tau \text{closeness}_i + \varepsilon_i
\]

Or:

\[
Score_i = \alpha + \beta \text{importance}_i + \gamma \text{vulnerability}_i + \delta \text{hub}_i + \theta \text{reverberation}_i + \varepsilon_i
\]

Interestingly, for all OSIIs scores and subscores (size, importance, complexity, interconnexions and OSI score), balance sheet and network centrality variables can explain around 50% of the variance, while our local measures perform better by explaining 90% or more of the variance, except for complexity. The hub effect alone explains 82% of the variance and has a positive coefficient while reverberation is the only measure to have a significant negative coefficient.

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\textsuperscript{41} EBA guidelines \texttt{EBA/GL/2014/10} “On the criteria to determine the conditions of application of Article 131(3) of Directive 2013/36/EU (CRD) in relation to the assessment of other systemically important institutions (O-SIIs)”. 

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